

## 4-6 Cramer's Rule

Gabriel Cramer

- used determinants to solve systems of equations

Use elimination to solve for x:

$$\begin{array}{r} d(ax + by = e) \\ -b(cx + dy = f) \end{array}$$

$$\begin{array}{r} adx + bdy = de \\ -bcx + -bdy = -bf \\ \hline (ad - bc)x = de - bf \\ x = \frac{de - bf}{ad - bc} \end{array}$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \quad \begin{vmatrix} e & b \\ f & d \end{vmatrix}$$

$$x = \frac{de - bf}{ad - bc}$$

$$\begin{array}{l} ax + by = e \\ cx + dy = f \end{array}$$

Denominator is:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Write a determinant matrix for the numerator:

Cramer's Rule:

$$x = \frac{D_x}{D} \quad y = \frac{D_y}{D}$$

Where  $D_x$  replaces the coefficients of x with the constants and  $D_y$  replaces the coefficients of y with the constants.

ex:

$$x - 3y = 5$$

$$2x + 9y = -10$$

$$D = \begin{vmatrix} 1 & -3 \\ 2 & 9 \end{vmatrix} = 15$$

$$D_x = \begin{vmatrix} 5 & -3 \\ -10 & 9 \end{vmatrix} = 15$$

$$D_y = \begin{vmatrix} 1 & 5 \\ 2 & -10 \end{vmatrix} = -20$$

$$x = \frac{15}{15} = \frac{D_x}{D}$$

$$x = 1$$

$$y = \frac{D_y}{D} = \frac{-20}{15} = -\frac{4}{3}$$

$$(1, -\frac{4}{3})$$

If  $D = 0$ 

then, either

 $\infty$  # of solutions

OR

 $\emptyset$ 

ex:

$$2x - y = 3$$

$$6x - 3y = -9$$

$2x - y = 3$   
 $2x - y = -3$  Not same eqn  
 therefore

$$D = \begin{vmatrix} 2 & -1 \\ 6 & -3 \end{vmatrix} = 0$$

$$D_x = \begin{vmatrix} 3 & -1 \\ -9 & -3 \end{vmatrix}$$

$$D_y = \begin{vmatrix} 2 & 3 \\ 6 & -9 \end{vmatrix}$$

3 Variables

Cramer's Rule:

$$x = \frac{D_x}{D} \quad y = \frac{D_y}{D} \quad z = \frac{D_z}{D}$$

ex:

$$4x + y - z = -2$$

$$x + 3y - 4z = 1$$

$$2x - y + 3z = 4$$

$$D = \begin{vmatrix} 4 & 1 & -1 \\ 1 & 3 & -4 \\ 2 & -1 & 3 \end{vmatrix} = 16$$

$$D_x = \begin{vmatrix} -2 & 1 & -1 \\ 1 & 3 & -4 \\ 4 & -1 & 3 \end{vmatrix} = -16$$

$$x = \frac{-16}{16} \quad y = \frac{96}{16} \quad z = \frac{64}{16}$$

$$x = -1$$

$$y = 6$$

$$z = 4$$

$$D_y = \begin{vmatrix} 4 & -2 & -1 \\ 1 & 1 & -4 \\ 2 & 4 & 3 \end{vmatrix} = 96$$

$$D_z = \begin{vmatrix} 4 & 1 & -2 \\ 1 & 3 & 1 \\ 2 & -1 & 4 \end{vmatrix} = 64$$

ex:

$$3x - 2y + 2z = 1$$

$$2x + 5y - 5z = 7$$

$$4x - 3y + z = -3$$

HW  
p 192-193  
12, 13, 28, 30